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Improved Methods for Large Scale Structural Synthesis

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Techniques for the improvement of procedures for large scale structural synthesis are investigated. One involves a method for control of oscillation found to occur in many optimization procedures. The other is a new primal mathematical programming formulation. The central idea for oscillation control is the use of the gradients of a potentially active constraint set to prevent serious violation of one of the set when a move is made considering only the active constraints. The mathematical programming procedure is based on an improved feasible direction finding formulation. Results of numerical experiments with these methods are very encouraging.

Nomenclature

A_i = unit weight of member i
C_1 = convergence specification
C_2 = minimum resizing parameter specification
E_{ij} = influence coefficients
e_j = bandwidth for jth constraint
$f(x_i)$ = objective function or mass
$g_j(x_i)$ = behavior constraints
I = number of design variables
J = behavior constraint set
K_η = resizing parameter reduction factor
K_e = potential bandwidth expansion factor
K_j = potential bandwidth factor
R_j = step reduction factor
R^* = min (R_j)
S_i = component of the move direction vector S
U_j = upper constraint limit
W_j = weighting parameter in direction finding problem
X = recursion relation
x_i = design variables
z_i = inverse variables $1/x_i$
α = step size
∇ = gradient operator
Δ = difference
η = resizing parameter
σ = dummy variable in the direction finding problem

Superscripts

ℓ = lower limit
p = preceding value
r = iteration or redesign number
T = matrix transform
u = upper limit

Subscripts

a = active constraint set
i = variable number
$,i$ = partial derivative with respect to the ith variable
j = constraint number
p = potentially active constraint set
v = constraint set expected to be violated

Introduction

MUCH recent effort has been devoted to structural optimization.[1] Yet as developments continue in this area, constant improvements in structural analysis methods, most of which are computationally demanding, tax the ability of existing optimization procedures to use the most powerful of the new analysis techniques effectively. Venkayya and his associates pioneered in the development of effective methods for large scale structural synthesis introducing the optimality criteria (OC) approach[2] for the optimization of structures modeled by finite elements. These efforts led to the development of automated design capabilities of significant value.[3] A somewhat more rigorous OC method for treating problems with multiple active constraints is described by Rizzi[4] and elaborated by Khot et al.[5,6]

Conventional mathematical programming (MP) techniques,[7] although relatively general and rigorous, are usually poorly suited to large scale structural synthesis due to the large number of reanalyses these methods typically required to produce convergence. Several investigators have, however, developed specialized MP methods using approximations,[8] penalty functions,[9] and dual methods[10] useful on such problems.

Among the problems encountered in varying degree with such procedures are divergence, convergence failure, lack of rigor, and relatively great computational effort associated with design resizing when multiple active constraints are present. This paper presents techniques for dealing with these problems. Approaches are presented for divergence and convergence controls. In addition, a new primal MP procedure based on an improved feasible direction (FD) formulation is described that provides a rigorous and yet apparently computationally efficient method for treating multiple active constraints.

Problem Statement

The structural optimization problem may be posed by the following.

Find

$$\min f(x_i) \qquad i=1,2...I \tag{1}$$

subject to the conditions

$$g_j(x_i) \le 0 \qquad j=1,2...J \tag{2}$$

and

$$x_i^\ell \le x_i \le x_i^u \tag{3}$$

For the minimum mass design of structures modeled by bar and membrane plate elements the equations may be given by[2]

$$f(x_i) = \sum_{i=1}^{I} A_i x_i \tag{4}$$

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$$g_j(x)=\left(\sum_{i=1}^{I}\frac{E_{ij}}{x_i}\right)-U_j\leq 0 \tag{5}$$

where the A_i and U_j are constants and E_{ij} implicit functions of the problem variables.

Resizing is generally accomplished in mathematical programming procedures by letting

$$x_i^{r+1}=x_i^r+\alpha^r(\eta)S_i^r \tag{6}$$

where the determination of the right-hand term of Eq. (6) often results from the solution of a resizing problem. The resizing parameter η and the $\alpha(\eta)$ function are usually selected arbitrarily. Optimality criteria methods resize the structure based on a solution of the problem: find x_i such that

$$f_{,i}+\sum_{j\subset J_a}\lambda_j g_{j,i}=0 \tag{7}$$

$$\lambda_j-g_j=0 \tag{8}$$

$$\lambda_j\geq 0 \tag{9}$$

$$j\subset J_a;\quad g_j>-e_{ja} \tag{10}$$

by methods of successive substitutions where a resizing problem is solved at each iteration and the structure resized by redefining x_i as

$$x_i^{r+1}=X(\eta,\lambda_j,x_i)^r \tag{11}$$

Oscillation and Divergence Control

Background

Almost all optimization methods suitable for large scale structural synthesis require the selection of an active constraint J_a set for inclusion into the resizing problem. This is usually done by specifying an arbitrary bandwidth of constraint activity as in Eq. (10). The inclusion of too many constraints often results in an unnecessary increase in the computational effort required for the solution of the resizing problem. This resizing effort can be substantial and may greatly exceed the reanalysis effort. In some procedures excess constraints may also "overconstrain" the problem producing heavier designs. On the other hand, selection of bandwidths that are too small may lead to the major violation of a constraint that was not included in the resizing problem, possibly resulting in a bad move.

A major difficulty in the use of optimization methods is that no rigorous, or even reasonably reliable, efficient procedure has been formulated for bandwidth selection. The desire to reduce the computational effort and avoid the overconstraint problem usually results in selection of narrow bandwidths and thus in occasional, or even frequent, problems with oscillation, divergence,[5,6] or failure to converge to the optimum. Oscillation or divergence resulting from such problems will be referred to here as "primary oscillation."

In addition to this mode, oscillation or failure to converge can also result from too large a value of the resizing parameter or step size.[5] This will be referred to as "secondary oscillation."

Procedure for Dealing with Primary Oscillation

The basic approach here is to introduce a potentially active constraint set J_p where

$$j\subset J_p;\quad -e_{ja}\geq g_j\geq -e_{jp} \tag{12}$$

$$e_{jp}=K_j e_{ja}\qquad K_j>1 \tag{13}$$

These constraints are not included directly in the resizing problem. However, if after resizing it appears, based on the gradient information associated with these constraints, that any of them may be violated the resizing step is shortened in an attempt to avoid this violation.

Where the new design x_j^{r+1} is given by Eq. (11). Call

$$\Delta x_i^r=x_i^{r+1}-x_i^r \tag{14}$$

The estimated value of g_j^{r+1} is given by

$$g_j^{r+1}=g_j^r+\sum_i g_{j,i}^r\Delta x_i^r\qquad j\subset J_p \tag{15}$$

If

$$g_j^{r+1}>0 \tag{16}$$

put these in a set of constraints J_v expected to be violated and compute a step reduction quantity R_j such that a move would produce a value of $g_j^{r+1}=0$ for these constraints. Thus,

$$R_j=g_j^r\Big/\left(\sum_i g_{j,i}\Delta x_i^r\right)\qquad j\subset J_v \tag{17}$$

Call

$$R^*=\min(R_j) \tag{18}$$

and redefine x_i^{r+1} as

$$x_i^{r+1}=x_i^r+R^*\Delta x_i^r \tag{19}$$

For equations of the form of Eqs. (4) and (5) where the E_{ij} are usually only weakly dependent on the variables, as is the case in finite element structures modeled by membrane type elements, greater precision in the estimates of the change in constraint values may be obtained by use of inverse variables $z_i=1/x_i$. Using such variables the control procedure may be restated as: where $g_j(z_i)$ is given by

$$g_j(z_i)=\left(\sum_i E_{ij}z_i\right)-U_j \tag{20}$$

if the redesign move is to $z_i^{r+1}=1/x_i^{r+1}$ where x_i^{r+1} is given by Eq. (11) and

$$\Delta z^r=z_i^{r+1}-z_i^r \tag{21}$$

then, if any

$$g_j^r+\sum_i E_{ij}^r\Delta z_i^r\geq 0\qquad j\subset J_p \tag{22}$$

put these in the set of constraints in set J_v expected to be violated and compute

$$R_j=-g_j^r/(E_{ij}^r\Delta z_i^r)\qquad j\subset J_v \tag{23}$$

and where R^* is given by Eq. (18) and redefine

$$\Delta z_i^r=R^*\Delta z_i^{rp} \tag{24}$$

Then the design variables at the next iteration are given by

$$x_i^{r+1}=1/[1/x_i^r+\Delta z_i^r] \tag{25}$$

Procedure for Dealing with Secondary Oscillation

Resizing parameter or step size reduction commonly is employed for oscillation control. Here such reduction is limited to the control of secondary oscillation so as not to slow convergence by use of needlessly small resizing moves.

Thus, if

$$f(x_i^{r+1}) > f(x_i^r) \tag{26}$$

and some constraint

$$g_j^{r+1} \subset J_a^{r+1} \tag{27}$$

and this constraint

$$g_j^{r+1} \not\subset J_p^r \text{ or } J_a^r \tag{28}$$

it is assumed that the resizing parameter must be reduced, and thus it is redefined by

$$\eta = \eta^p / K_\eta \tag{29}$$

If Eq. (28) is not satisfied it is assumed that the potential bandwidth was too narrow and the potential bandwidth factor associated with this constraint is redefined as

$$K_j = K_e K_j^p \qquad K_e > 1 \tag{30}$$

In both these cases a new design move is then computed at point $x_i^r = x_i^{rp}$ using the new resizing parameter or bandwidth factor.

Termination Procedure

The process is terminated by a conventional MP procedure. If

$$f(x^{r+1}) - f(x^r)/f(x^r) \le C_1 \tag{31}$$

or if

$$\eta < C_2 \tag{32}$$

the optimization procedure is stopped.

Added Storage Requirements and Computational Effort

Once a step reduction value R_j is computed all the gradient information associated with that constraint may be discarded. Thus it is only necessary to store one set of constraint gradient components rather than the gradients of all constraints in the potentially active set. Furthermore, in many procedures once the quantities associated with the solution of the resizing problem are computed the gradients of the active set are no longer needed. Some part of the active constraint gradient storage, therefore, may be used for the particular potentially active constraint being considered. Thus, no additional storage is required for the primary oscillation control procedure in such instances.

The calculation of the additional potentially active constraint gradients requires additional computational effort at each redesign cycle. This additional effort, however, is usually small compared to the total computational effort and usually smaller than the effort wasted by bad moves resulting from failure to consider, at all, these potentially active constraints.

In the case of methods using the secondary oscillation control procedure, if one is to formulate a new resizing problem at a point x_i^r using a smaller η, computational effort may be reduced if information associated with the former resizing problem is saved. This information generally is derived from the gradients of the active constraint set. Since active constraint set gradient storage is no longer needed this resizing problem information may be stored in its place. Furthermore, since the active constraint set gradients usually require much more storage than the resizing information this extra storage may be used for some or all of the potentially active constraint gradients. Thus, with appropriate information storage these procedures should not normally require a significant increase in storage capacity.

Feasible Direction MP Procedure

The feasible direction problem is usually formulated[7] as follows. Find S_i and σ so as to

$$\text{maximize } \sigma > 0 \tag{33}$$

where

$$S^T \nabla f(x_i) + \sigma \le 0 \tag{34}$$

$$S^T \nabla g_j(x_i) + W_j \sigma \le 0 \qquad j \subset J_a \tag{35}$$

$$-1 \le S_i \le 1 \tag{36}$$

Vector S consisting of components S_i is the best "feasible direction" of movement.

Equations (33) and (34) state that on the basis of linearized functions the solution to this problem will produce a maximum possible improvement in $f(x_i)$. Equations (33) and (35) state for $W_j = 0$ there will be no movement toward constraint violation, and where $W_j > 0$ there will be movement away from violation. Equation (36) is used to eliminate unbounded solutions. This is a linear programming problem and may be solved efficiently by one of many well-developed procedures.

The weighting parameter in most feasible directions methods is given as positive in order to avoid constraint violation. Such violation will occur on the convex constraints that are encountered in structural design. Here one has the problem of determining an appropriate active constraint set to include in Eq. (35). Too large a bandwidth will overconstrain the problem by forcing movement essentially parallel to, or away from, a constraint that may not yet be critical. Too small a bandwidth, on the other hand, may produce serious violation of a constraint that was not considered in the direction finding problem of Eqs. (33-36).

The central idea of the improved formulation is to set weighting such that it will produce movement toward active but not yet critical constraints. This eliminates the overconstraining effort of too large a bandwidth. Thus, replace the $W_j\sigma$ term in Eq. (35) with a term producing movement toward the constraint and thereby rewrite Eq. (35) as

$$\alpha^r S^T \nabla g_j^r(x_i) \le g_j^r \tag{37}$$

It may be seen that the left-hand side will produce an estimated change at most equal to the constraint value. Then, after the resizing move of αS the value of the constraint will move toward zero, or even near zero, if the constraint is critical. In other words, if movement toward violation of the constraint improves the design, Eq. (37) will limit that movement only to the extent required to avoid violating the constraint. Thus, the redesign move will be influenced only by constraints expected to be critical.

Regional constraints may be included by rewriting Eq. (36) as

$$S_i^\ell \le S_i \le S_i^u \tag{38}$$

where

$$\alpha S_i^\ell = x_i - x_i^\ell, \qquad x_i - x_i^\ell < 1$$
$$= 1, \qquad x_i - x_i^\ell \ge 1 \tag{39}$$

$$\alpha S_i^u = x_i^u - x_i, \qquad x_i^u - x_i < 1$$
$$= 1, \qquad x_i^u - x_i \ge 1 \tag{40}$$

Finally, the problem may be put into standard linear programming form by the transformation

$$(S_i)' = S_i + S_i^\ell \tag{41}$$

Then one has the transformed problem

$$\text{maximize } \sigma > 0 \qquad (42)$$

$$(S^T)' \nabla f + \sigma \leq \sum_i \nabla f_i \qquad (43)$$

$$(S^T)' \nabla g_j \leq g_j / \alpha - \sum_i \nabla g_{ji} \qquad (44)$$

$$0 \geq (S_i)' \leq S_i^{\ell} + S_i^u \qquad (45)$$

The upper bounds of these problem variables may be handled by a bookkeeping procedure and they do not expand problem size.[11] Where the right-hand side of Eq. (43) or (44) is negative it may be converted to standard form by reversing the sign of the equation and the limit sign.

It should be noted that this procedure may also produce oscillation or divergence resulting from deficiencies in bandwidth or η selection. The same procedures described earlier for control of oscillation and divergence may of course be used here.

The value for α^r for this study is given by

$$\alpha^r = \eta f(x_i^r) / (\mid S^r \mid / \nabla f^r) \qquad (46)$$

where $\mid S^0 \mid = \sqrt{I}$ and η is arbitrarily selected. Such a value of η will tend to produce a change in the objective function of approximately $100\eta^r$% if a move, S^r, is made in the ∇f direction. Thus, $\eta = 0.5$ would tend to produce about a 50% change in weight if one so moved. Since a move defined by the FD problem would be deflected away from ∇f by the constraints, the change after such a move would be substantially less. This would be particularly true during the later stages of the search, where the design is highly constrained.

The $\mid S^r \mid$ is estimated using the magnitude of the previous S and solving the FD linear programming problem. The magnitude of this direction is then used in Eq. (46) to calculate the move. Since only the right-hand side of Eq. (44) are changed, the move direction is easily computed without resolving the linear programming problem using the existing linear programming optimal basis.[11]

If a redesign move fails to produce a weight reduction, η is reduced, design x^{r+1} is discarded, and a new design is generated from point x_i^r. When this occurs the bandwidths are also narrowed by setting

$$e_j = e_j / 2 \qquad (47)$$

The initial e_j are arbitrarily selected.

The design is restored to the feasible-infeasible boundary by scaling as in Ref. 2 or by some other procedure where the problem is not of the form of Eqs. (4) and (5).[12]

Termination is accomplished by the aforementioned procedure or if

$$\mid S^r \mid \leq C_1 \qquad (48)$$

which, when C_1 is sufficiently small, indicates a local optimum has been located.

Numerical Experiments

Problems

The two ten bar truss problems posed by Venkayya et al.[2] are repeated in this study using the parameters of Ref. 5. The "stress" constrained problem involves the minimum weight design of an indeterminate structure under a single loading condition where the stress in all members must be held at or below one of two specified values. The optimal design in this structure has a mass of 679.30 kg. In the "displacement" constrained problem two additional constraints are placed on the deflection of two joints and only one stress limit value is used. Regional constraints [Eq. (3)] are used for all variables. The stresses and deflections are determined by a finite element analysis the results of which may be used to develop the constraint gradient information by means of the virtual unit load method.[2]

These are now classic benchmark problems and represent a difficult challenge for any proposed structural optimization scheme. The stress constrained problem has eight (of ten) active stress constraints in addition to several active regional constraints. The displacement constrained problem has two local minima, one of which is constrained by the two displacement constraints (this design has a mass of 2302.75 kg) and one by a stress and one displacement constraint (this design has a mass of 2295.63 kg). Several regional constraints are critical at both local optima.

Procedures

The generalized OC procedures described by Khot et al.[5,6] as the exponential [Eq. (7) of Ref. 6], and linear [Eq. (9) of Ref. 6] recursion forms were modified to use the oscillation control techniques described herein. In addition, this procedure was also adapted to use the multiple λ iteration Newton-Raphson procedure described by Khot et al. in Ref. 5. The procedure of Ref. 5 is similar to that of Ref. 4 except for the method of solution of the λ problem. The oscillation control methods were added to the experimental program used to obtain the results given in Refs. 5 and 6 to generate the results contained herein.

The MP program used here was also developed from this same experimental program by replacing the resizing portions of the program. It did not employ the primary oscillation control technique.

Results of Use of Oscillation and Divergence Control Procedure

Primary Oscillation Control

Table 1 illustrates the application of this procedure to two cases where serious oscillation was experienced using the OC procedures of Refs. 5 and 6. Both involve the stress constrained problem. In the first case the exponential recursion form with multiple iterations solution for the λ set was employed, and in the other the exponential form with single solution for the λ set is used.[5] The step size η used here is the

Table 1 Design sequence, stress constrained 10-bar truss problem with and without primary oscillation control; $\eta = 0.5$, mass in kg

	Linear form multiple λ iterations		Exponential form single λ solution	
Design No.	No control	Control	No control	Control
1	1558	1558	1558	1558
2	2019	934	1414	984
3	1062	889	2209	903
4	3275	817	1274	1040
5	3890	755	1246	834
6	1220	750	2202	851
7	754	699	1288	763
8	1011	695	934	751
9	773	692	984	724
10	738	690	1350	714
11	10,883	688	1756	707
12	5073	686	1347	702
13	2129	684	1529	698
14	4397	683	1268	695
15	2244	681	1969	692
16	6417	679.9	1132	689
17	3136	679.5	1760	687
18	3174	695	1225	685
19	4878	679.3	1871	684
20	2571	679.5	1040	682
25	–	679.3	–	759

Table 2 Design sequence, stress-constrained 10-bar truss problem single λ solution, linear form $\eta = 0.5$ at start, mass in kg

Design No.	Primary control only	Both controls
1	1558	1558
2	1109	1109
3	842	842
4	1041	1041[a]
5	845	783
6	935	750
7	783	719
8	779	706
9	712	698
10	699	694
11	691	691
12	689	689
13	686	686
14	684	684
15	683	683
16	682	682
17	681	681
18	680	680
19	679.30	679.36
20	751	751[a]
21	764	714[a]
22	762	696[a]
23	761	688[a]
24	761	679.33
25	761	683[a]
26		679.31
27		681[a]
28		679.30

[a] Halved resizing parameter at this redesign cycle.

Table 3 Comparison of design sequences using different initial resizing parameters, stress-constrained 10-bar truss, mass in kg

Design No.	Multiple λ iterations $\eta = 1$ at start	Multiple λ iterations $\eta = 0.5$ at start	Single λ solution $\eta = 1$ at start	Single λ solution $\eta = 0.5$ at start
1	1558	1558	1558	1558
2	965	934	1243	1109
3	689	886	828	833
4	804	817	780	1041[a]
5	743	755	738	783
6	723	750	717	750
7	695	699	702	719
8	693	695	694	705
9	690	692	689	698
10	686	690	684	694
11	684	688	680	691
12	683	686	688[a]	689
13	680	684	742[a]	686
14	707[a]	683	679.44	684
15	679.31	681	713[a]	683
16	679.30	680	681[a]	682
17	679.30[b]	679.5	679.34	681
18		695[a]	688[a]	680
19		679.31	683[a]	679.5
20		679.30	683[a,c]	751[a]
21		679.30[b]		713[a]
22				696[a]
23				688[a]
24				679.33
25				683[a,c]

[a] Halved resizing parameter. [b] Terminated by convergence specification. [c] Terminated by minimum resizing parameter specification.

reciprocal of the exponent and relaxation parameters used in the exponential and linear recursion forms of Refs. 5 and 6. It may be seen that the use of this procedure did, in fact, control primary oscillation.

Two other test cases were used in which the OC procedures described in Refs. 5 and 6 were induced to oscillate badly by use of an excessively large resizing parameter ($\eta = 1$). The use of the primary oscillation control again suppressed this oscillation mode.

Table 4 Comparison of design sequences using different initial resizing parameters, displacement constrained 10-bar truss, mass in kg

Design No.	Multiple λ iterations $\eta = 1$ at start	Multiple λ iterations $\eta = 0.5$ at start	Single λ solution $\eta = 1$ at start	Single λ solution $\eta = 0.5$ at start
1	3749	3749	3749	3749
2	3127	3015	3241	3024
3	2638	2642	3144	2810
4	2537	2587	3012	2698
5	2479	2539	2439	2612
6	2416	2482	2368	2553
7	2430	2428	2366	2502
8	2497[a]	2356	2341	2450
9	2359	2304	2349[a]	2395
10	2311	2304	2326	2353
11	2302	2306[a]	2321	2333
12	2298	2302.89	2320	2320
13	2295.63	2302.75	2319	2317
14	2295.57	2302.75[b]	2319	2309
15	2295.56		2321[a]	2315[a]
16	2295.56[b]		2318.54	2311
17			2318.47	2309
18			2318.45	2308
19			2318.42	2307.4
20			2318.40	2307.4
25			2312	2306.9[a]
30			2311	2306.9[a]
35			2310.33	2305.6[b]
50			2310.28	

[a] Halved resizing parameter. [b] Terminated by convergence specification.

Secondary Oscillation Control

Table 2 illustrates the application of the secondary procedure. With only primary control one sees here secondary oscillation at design numbers 4-6 and divergence at design number 20. It may be seen that the reduction in the resizing parameter produces convergence to the optimal design.

This procedure was found satisfactory only with the linear recursion forms. It was found unsatisfactory for the exponential form since the assumption used in Refs. 5 and 6 that the λ problem formulated on the basis of the linear recursion forms may be used with the exponential form is not satisfactory at small values of η.

All runs used $K_a = K_\eta = K_e = 2$. The quantities e_j for the active constraints are defined by the procedure of Ref. 6. Termination constants used for all runs were $C_1 = 10^{-5}$, $C_2 = 0.01$.

Use of a Large Initial Resizing Parameter

Since the primary control experiments indicated that these controls would inhibit oscillation that would usually occur with excessive values of η, an experiment was performed to investigate the possibility of using a large initial value of this parameter so as to speed convergence. The result is shown in Tables 3 and 4. There seems to be a minor advantage in the use of large η values on the easier stress constrained problem but a large η is disadvantageous on the more difficult displacement constrained problem.

Convergence of Multiple and Single λ Problem Solution Procedures

It may also be seen from Tables 3 and 4 that the multiple λ iteration procedure possesses much better convergence properties than the single solution procedure. In fact, the single λ solution procedure fails on the displacement constrained problem. It also fails on this problem when no oscillation controls are used.

Table 5 Comparison of inverse and ordinary variables using multiple λ iteration linear form procedure, $\eta = 0.5$ at start, mass in kg, 10-bar truss.

Design No.	Stress constrained Inverse	Stress constrained Ordinary	Displacement and stress constrained Inverse	Displacement and stress constrained Ordinary
1	1558	1558	3749	3749
2	952	934	3135	3014
3	863	889	2920	2642
4	754	817	2810	2587
5	737	755	2730	2539
6	967	750	2666	2482
7	689	699	2608	2428
8	686	695	2553	2356
9	684	692	2500	2304.2
10	683	690	2450	2303.8
11	681	688	2407	2306[a]
12	723[a]	686	2351	2302.89
13	679.33	684	2314	2302.75
14	679.30	683	2311	2302.75
15	740[a]	681	2305	
16	679.30[b]	679.84	2311[a]	
17		679.44	2305	
18		695[a]	2302.89	
19		679.31	2302.75	
20		679.30	2316[a]	
21		679.30[b]	2302.75[b]	

[a] Halves resizing parameter. [b] Terminated by convergence specification.

Table 6 Design sequences for feasible direction algorithm, mass in kg, 10 bar truss

Design No.	η, Stress constrained 0.1	0.3	0.5	η, displacement and stress constrained 0.2
1	1558	1558	1558	3749
2	1429	1219	1096	3345
3	1300	967	1208[a]	3095
4	1197	863	894	3001
5	1112	944[a]	813	2908
6	1037	877	738	2763
7	968	808	713	2705
8	903	775	771[a]	2568
9	845	756	735	2470
10	806	735	697	2345
11	802	722	694	2885[a]
12	755	710	688	2328
13	731	701	679.48	2345[a]
14	715	694	679.30	2302
15	702	683	679.30[b]	2331[a]
16	684	679.30		2298
17	679.34	679.30[b]		2296.54
18	679.30			2296.08
19	679.30[b]			2295.90
20				2295.58
25				2295.57[b]

[a] Halved resizing parameter. [b] Terminated by convergence specification.

An oscillation control procedure is important in allowing exploitation of the multiple λ iteration approach because of the oscillation problems with this method encountered in earlier studies.

Comparison of Ordinary and Inverse Variables

The primary oscillation control method used the inverse variable form in the computations involving violation estimates and step reduction since it seems clear that their use then would provide better estimates. The OC programs tested were also modified to use inverse variables in an effort to see if their use in the solution of the resizing problem was also advantageous. Here the superiority of this form is not clear since the objective function becomes nonlinear. The effect on convergence by use of inverse variables in the resizing problem thus was studied. The results of this study are shown in Table 5. There is no apparent substantial advantage associated with the use of inverse variables in the resizing problem formulation. The inverse form is superior on the easier stress constraint problem but inferior on the displacement problem.

200 Bar Truss Example

The abovementioned control procedures were also adapted to Khots experimental program used to generate the results for his 200 bar truss example.[6] All cases where oscillation was reported in Ref. 6 were run using this modified procedure. Here again oscillation and divergence due to primary and secondary causes was eliminated.

Performance of the FD Procedure

Some of the results of these procedures applied to the two 10 bar truss examples are shown in Table 6. All cases use initial $e_{ja} = 0.5U_j$. The effect of varying the step size parameter may be seen by examining the results of the stress constrained problem. Runs were also made with initial $\eta = 0.2$ and 0.4 with termination of the optimum after 16 and 14 cycles, respectively. It may be seen that the number of cycles required to reach the optimum is relatively insensitive to step size. Smaller step size produce smaller initial weight changes but waste fewer analyses by bad moves resulting from the need for step size reduction.

A similar situation exists for the displacement and stress constrained problem where runs were made at similar values of η. All produced convergence within 22-30 analyses. A typical run is shown in Table 6.

The modified FD formulation of Eqs. (41-45) was found to be substantially more effective than the conventional form of Eqs. (33-36). A program identical to that generating the results previously described but employing the conventional FD form was also tested. The computational effort required by this procedure to reach the optimum was typically an order of magnitude greater than required by the procedure using the modified FD form.

Table 7 Typical resizing to analysis time ratios

	10-bar truss, stress constrained	10-bar truss stress and displacement constrained	200-bar truss[5]
Multiple λ iterations	90	17	7.4
Single λ solution	6	3.2	0.3
Feasible directions	3.5	2.1	–

Comparison of Procedures

A qualitative estimate of the effectiveness of the various procedures studied here may be obtained by a comparison of their performance on the test problems. On the basis of this limited comparison one finds that the FD procedure appears to provide substantially better convergence properties than singe λ solution OC procedure as well as convergence comparable, although perhaps somewhat inferior, to the multiple λ iteration OC procedure but only after the oscillation controls have been added to the OC methods. One cannot, however, base such comparisons strictly on number of analyses to convergence. If one compares these procedures on the basis of total computational effort the MP procedure appears to be more efficient than the OC methods studied here.

Consider the typical resizing to analyses time ratios shown in Table 7. The analysis time is that required to solve the finite element problem. The resizing effort includes the time required for computation of the required derivatives and for the solution of the λ set or FD finding problem. It may be seen that the resizing effort associated with the FD procedure is comparable to that required by the single λ solution OC method. With these methods the bulk of the resizing time is associated with derivative computation. The multiple λ iteration method, however, requires substantially more resizing effort, the bulk of which is associated with the solution of the λ set problem. Thus the attractiveness of the multiple λ iteration approach disappears due to the excessive resizing effort required. It may be seen that a comparison must, therefore, consider both resizing effort and convergence if it is to be meaningful as a measure of algorithm effectiveness.

Conclusion

More work needs to be done to verify the results developed here and to further refine the concepts presented. The results of this work, however, support the assumptions that oscillation problems associated with some optimization methods may be controlled effectively by the procedure described herein. Further, it appears a simple primal MP procedure without use of approximations may be comparable with OC procedures for finite element based structural synthesis.

The MP procedure seems particularly attractive on problems with many active constraints due to the efficiency of the linear programming procedures in solving the resizing problems and the rigor of this approach. The ability of the OC procedures to produce very large initial weight reductions makes these procedures attractive on problems with few active constraints. Additional work needs to be done to reduce the resizing computational effort of the multiple λ iteration approach to allow exploitation of its superior convergence properties.

No effort was made to "tune" the algorithm control constraints such as K_η, K_e, K_j, e_j, etc. In all cases the number 2 or its reciprocal were arbitrarily chosen simply as reasonable values. No values other than those reported were tried. Thus, these results may be considered typical of those that might be achieved in general use for these procedures in their existing form. It should be noted that the FD method described here is at an early stage in its development, and that no attempt was made to exploit the particular properties of the problem types studied. It is reasonable to expect further improvements in this approach.

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